

Jai More · Anuradha Misra

Application of Coherent State Approach for the cancellation of Infrared divergences to all orders in LFQED

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Abstract We sketch an all order proof of cancellation of infrared (IR) divergences in Light Front Quantum Electrodynamics (LFQED) using a coherent state formalism. In this talk, it has been shown that the true IR divergences in fermion self energy are eliminated to all orders in a light-front time-ordered perturbative calculation if one uses coherent state basis instead of the usual Fock basis to calculate the Hamiltonian matrix elements.

Keywords Coherent state Formalism · Light front QED

1 Introduction

In the LSZ formalism, the asymptotic states are taken as free states and the S-matrix elements are the residues of the poles that arise in the Fourier transform of the correlation functions when four-momenta of the external particles are put on-shell. Therefore, the initial and final states used to calculate the transition matrix elements are taken to be the Fock states. However, in Quantum Electrodynamics (QED) the asymptotic states are not free states and the fermion is accompanied by soft photons i.e. photons with very low momentum. In an actual experiment, due to the finite size of the detector, the charged particle can be accompanied by any number of such photons, which are source of Infrared (IR) divergences. In the soft photon limit, the virtual and real emission are indistinguishable. So when we are dealing with a virtual photon correction in a process, we need to take into account emission of an infinite number of real soft photons also. Hence, the physical state should be defined as a set of states with an infinite number of soft photons.

A method of asymptotic dynamics was developed by Kulish and Faddeev (KF) to address the issue of cancellation of IR divergences at amplitude level [1]. They were the first to show that in QED, the asymptotic Hamiltonian does not coincide with the free Hamiltonian. KF constructed the asymptotic Hamiltonian V_{as} for QED thus modifying the asymptotic condition to introduce a new space of asymptotic states.

KF further modified the definition of S-matrix and showed that it is free of IR divergences. Thus the method of asymptotic dynamics proposed by KF replaces, the free Hamiltonian by an asymptotic Hamiltonian which takes into account the long range interaction between incoming and outgoing states and can be used to construct a set of coherent states as the asymptotic states. The transition matrix elements formed by using these states are then IR finite.

Jai More
Department of Physics, University of Mumbai, SantaCruz(East), Mumbai-400098, India.
E-mail: more.physics@gmail.com

Anuradha Misra
Department of Physics, University of Mumbai, SantaCruz(East), Mumbai-400098, India.
E-mail: anuradha.misra@gmail.com

In Light Front Field Theory (LFFT), there are two kinds of IR divergences viz. the *true* IR divergences which are the bona-fide divergences of equal-time field theory and which appear when both k^+ and \mathbf{k}_\perp approach zero and the *spurious* IR divergences that are just a manifestation of ultra-violet divergences of equal-time theory. The KF method has been used by various authors [2; 5; 6; 7] in the context of equal time theories. In LFFT, a coherent state formalism was developed in Ref. [8; 9; 10] as a possible method to deal with the *true* IR divergences in one loop vertex correction for LFQED and LFQCD. It has also been shown that the IR divergences in the fermion self-energy correction at the two-loop order cancel in LFQED [11; 12] if one uses the coherent state approach.

In QED, a general approach to treat IR divergences was given by Yennie et al. [13]. Following the work of Yennie et al., Chung [14] showed that IR divergences indeed cancel to all orders in perturbation theory at the level of amplitude. KF approach was applied by Greco et al [6] to study the IR behavior of non abelian gauge theories using coherent states of definite color and factorized in fixed angle regime. We shall present a general proof of all order cancellation of IR divergences in fermion mass renormalization in LFQED by using the coherent state method. The details of the calculation can be found in Ref.[15].

2 Coherent State Formalism in Light Front Field Theory

The coherent state method is based on the observation that for theories with long range interactions or theories having bound states as asymptotic states, the total Hamiltonian does not reduce to the free field Hamiltonian in the limit $|t| \rightarrow \infty$. In LFFT, the asymptotic Hamiltonian H_{as} , is evaluated by taking the limit $|x^+| \rightarrow \infty$ of the interaction Hamiltonian. Each term in the interaction Hamiltonian H_{int} has a light-cone time dependence of the form $\exp[-i(p_1^- + p_2^- + \dots + p_n^-)x^+]$ and therefore, if $(p_1^- + p_2^- + \dots + p_n^-)$ vanishes at some vertex, then the corresponding term in H_{int} does not vanish in large x^+ limit. Thus, the total Hamiltonian can be written as

$$H = H_{as} + H'_I \quad (1)$$

where

$$H_{as}(x^+) = H_0 + V_{as}(x^+) \quad (2)$$

In the Schrödinger representation, light cone time evolution operator $U_{as}(x^+)$ satisfies

$$i \frac{dU_{as}(x^+)}{dx^+} = H_{as}(x^+)U_{as}(x^+) \quad (3)$$

and can then be used to generate an asymptotic space

$$\mathcal{H}_{as} = \exp[-\Omega^A(x^+)]\mathcal{H}_F \quad (4)$$

from the usual Fock space \mathcal{H}_F , in the limit $x^+ \rightarrow -\infty$, where $\Omega^A(x^+)$ is the asymptotic evolution operator defined by

$$U_{as}(x^+) = \exp[-iH_0x^+] \exp[\Omega^A(x^+)] \quad (5)$$

The coherent states is defined as

$$|n : coh\rangle = \exp[-\Omega^A]|n\rangle \quad (6)$$

KF proposed the method of asymptotic dynamics [1] in the context of equal time QED wherein the asymptotic Hamiltonian is not just the free Hamiltonian but also contains the large time limit of those terms in interaction Hamiltonian which do not vanish at infinitely large times. This Hamiltonian is then be used to construct the asymptotic Möller operator and the coherent states. The true IR divergences corresponding to k^+ , $\mathbf{k}_\perp \rightarrow 0$ are expected to disappear when one uses this coherent state basis to calculate the transition matrix elements. The light-cone time dependent interaction Hamiltonian is given by

$$H_I(x^+) = V_1(x^+) + V_2(x^+) + V_3(x^+)$$

where [8]

$$V_1(x^+) = e \sum_{i=1}^4 \int d\nu_i^{(1)} [e^{-i\nu_i^{(1)}x^+} \tilde{h}_i^{(1)}(\nu_i^{(1)}) + e^{i\nu_i^{(1)}x^+} \tilde{h}_i^{(1)\dagger}(\nu_i^{(1)})] \quad (7)$$

$\tilde{h}_i^{(1)}(\nu_i^{(1)})$ are three point QED interaction vertices. At asymptotic limits, non-zero contributions to $V_1(x^+)$ come from regions where $\nu_i^{(1)} \rightarrow 0$. It is easy to see that $\nu_2^{(1)}$ and $\nu_3^{(1)}$ are always non-zero and therefore, \tilde{h}_2 and \tilde{h}_3 do not appear in the asymptotic Hamiltonian. Thus, the 3-point asymptotic Hamiltonian is defined by the following expression [8]

$$V_{1as}(x^+) = e \sum_{i=1,4} \int d\nu_i^{(1)} \Theta_\Delta(k) [e^{-i\nu_i^{(1)}x^+} \tilde{h}_i^{(1)}(\nu_i^{(1)}) + e^{i\nu_i^{(1)}x^+} \tilde{h}_i^\dagger(\nu_i^{(1)})] \quad (8)$$

where $\Theta_\Delta(k)$ is a function which takes value 1 in the asymptotic region and is zero elsewhere. The detailed discussion of construction of coherent state basis can be found in Ref. [8; 11; 12]. The asymptotic states can be defined in the usual manner by

$$|n: coh\rangle = \Omega_\pm^A |n\rangle, \quad (9)$$

where $|n\rangle$ is a Fock state and Ω_\pm^A , the asymptotic Möller operator is defined by

$$\Omega_\pm^A = T \exp \left[-i \int_{\mp\infty}^0 [V_{1as}(x^+) + V_{2as}(x^+)] dx^+ \right] \quad (10)$$

where $V_{2as}(x^+)$ is the asymptotic limit of the four-point instantaneous interaction terms of LFQED Hamiltonian. The strategy to construct an all order proof of the cancellation of IR divergences in the fermion self energy correction using the coherent state basis is based on the method of induction[15].

3 All order proof of cancellation of Infrared divergences in LFQED

The all order proof of cancellation of IR divergences in LFQED is based on the method of induction [13] and involves the following steps:

- I We first show the cancellation of IR divergences up to $O(e^4)$ in LFQED [11; 12] in our new graphical notation.
- II We assume that IR divergences cancel up to $O(e^{2n})$. We express this IR finite $O(e^{2n})$ contribution graphically as in Fig.1.
- III Then we express the $O(e^{2(n+1)})$ contribution in terms of IR finite $O(e^{2n})$ matrix elements by adding virtual photon lines for Fock state diagrams and real photon lines for coherent state diagrams
- IV Finally we show that these additions to $O(e^{2n})$ diagrams conspire to cancel the IR divergences up to $O(e^{2(n+1)})$ between the real and virtual diagrams.

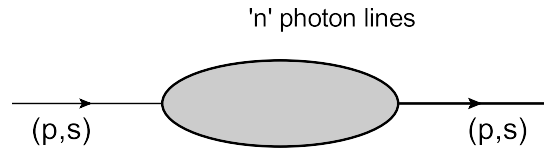


Fig. 1 IR finite blob representing the sum of all $O(e^{2n})$ diagrams in coherent state basis

We give below a brief description of the proof. The details of the proof are in Ref. [15]. The general expression for transition matrix element in $O(e^{2n})$ is a sum of terms of the form:

$$T_j^{(n)} = -\frac{e^{2n}}{2p^+(2\pi)^{3n}} \int \prod_{i=1}^n \frac{d^3 k_i}{2k_i^+ 2p_{2i-1}^+} \quad (11)$$

$$\times \frac{\bar{u}(\vec{p}, s_1) \not{\epsilon}_1 (\not{p}_1 + m) \not{\epsilon}_2 (\not{p}_2 + m) \cdots (\not{p}_i + m) \not{\epsilon}_i u(p, s_i)}{\prod_r (p^- - p_r^- - \sum_i k_i)} \quad (12)$$

We express the total $O(e^{2n})$ contribution as

$$T^{(n)} = \sum_j T_j^{(n)} = \sum_j \frac{\bar{u}(\bar{p}, s') \mathcal{M}_n^{(j)} u(p, s)}{\mathcal{D}^{(j)}} \quad (13)$$

where j is summed over all possible diagram in $O(e^{2n})$ and will be assumed to be IR divergence free.

$$\mathcal{D}^{(j)} = \prod_{i=1}^n \mathcal{D}_i^{(j)} \quad (14)$$

$\mathcal{M}_n^{(j)}$ is represented graphically by the blob in Fig 1. We have already given the proof of cancellation of IR divergences in δm^2 up to $O(e^4)$ in Ref. [11]. Now, we revisit this proof in the new graphical notation to make our strategy more lucid. Consider the $O(e^4)$ corrections which are represented by the two diagrams on the rhs of Fig. 2. It has been shown in Refs. [11; 12] that the sum of these two is IR finite.

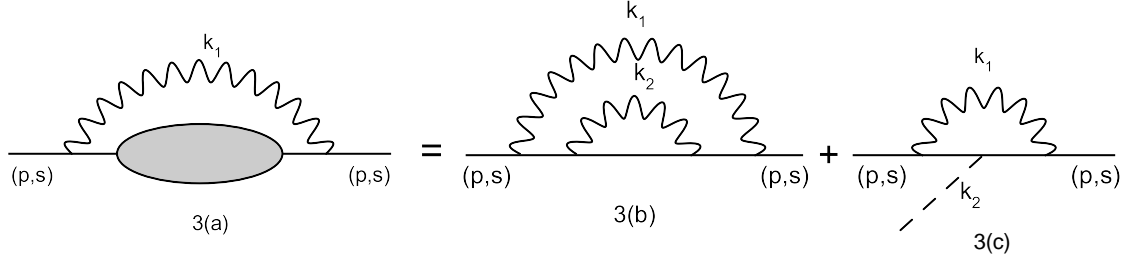


Fig. 2 IR finite $O(e^2)$ blob with an external photon line results in a sum of $O(e^4)$ diagrams

In our new notation

$$\begin{aligned} T_{3a}^{(2)} &= T_{3b}^{(2)} + T_{3c}^{(2)} \\ &= \frac{e^2}{(2\pi)^3} \int \frac{d^3 k_1}{2k_1^+} \frac{\bar{u}(p, s) \not{\epsilon}(k_1) (\not{p}_1 + m) \mathcal{M}_2^{(j)} (\not{p}_1 + m) \not{\epsilon}(k_1) u(p, s)}{(p \cdot k_1)^2 \mathcal{D}^{(j)}} \end{aligned} \quad (15)$$

where $\mathcal{M}_2^{(j)}$ represents the IR finite blob of Fig.1 without the external lines.

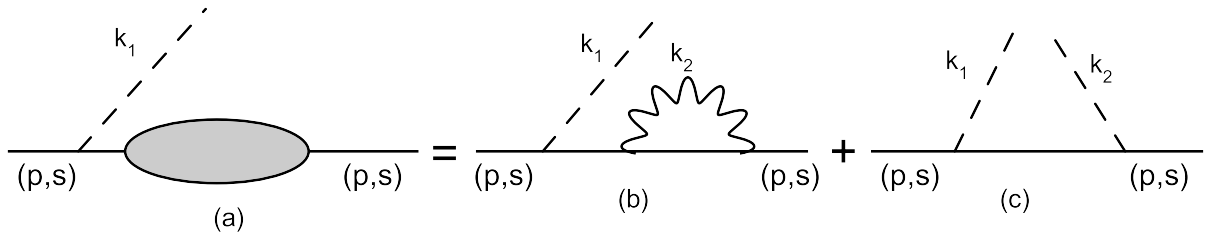


Fig. 3 IR finite $O(e^2)$ blob with an external photon line results in a sum of $O(e^4)$ diagrams in coherent state basis

Fig. 3 represents the additional $O(e^2)$ diagrams in coherent state basis which cancel IR divergences in Fig. 2. The additional contribution due to these in our new notation is

$$T_{4a}^{(2)} = -\frac{e^2}{(2\pi)^3} \int \frac{d^3 k_1}{2k_1^+} \frac{\bar{u}(p, s) \mathcal{M}_2^{(j)} (\not{p}_1 + m) \not{\epsilon}(k_1) u(p, s) (p \cdot k_1)}{(p \cdot k_1)^2 \mathcal{D}^{(j)}} \quad (16)$$

In the limit $k^+ \rightarrow 0, k_\perp \rightarrow 0$ the sum of the terms on rhs of Eqs. (15) and (16) is IR finite. A similar argument can be constructed for the remaining two diagrams which are graphically represented by Fig. 4.

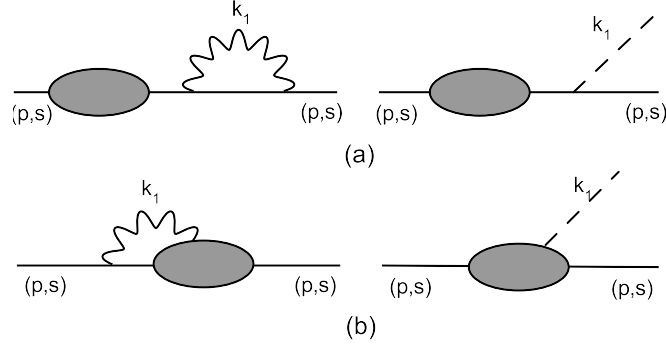


Fig. 4 Additional diagrams corresponding to $O(e^4)$ contributions in Fock and coherent state bases.

Now, we consider an $O(e^{2n})$ blob which we will assume to be free of IR divergences as shown in Fig. 1. We shall show that the cancellation of IR divergences in the $O(e^{2(n+1)})$ contribution to fermion mass renormalization in LFQED follows from the assumption that $O(e^{2n})$ diagrams are free of IR divergences. To construct an $O(e^{2(n+1)})$ diagram in Fock basis, we can add a photon to n^{th} order blob in three different ways as shown in Fig. 5.

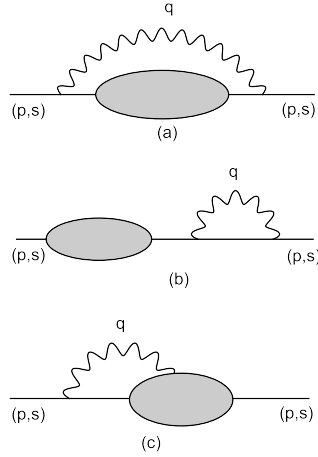


Fig. 5 Addition of a photon line to $O(e^{2n})$ blob in Fock basis

The contributions coming from the diagram in Figs. 5(a), (b) and (c), in the limit $q^+ \rightarrow 0$, $\mathbf{q}_\perp \rightarrow 0$ are given by

$$T_{5a}^{(n+1)} = \frac{e^2}{(2\pi)^3} \int \frac{d^3 q}{2q^+} \frac{\bar{u}(p, s) \not{\epsilon}(q) (\not{P} + m) \mathcal{M}_n^{(j)}(\not{P} + m) \not{\epsilon}(q) u(p, s)}{(p \cdot q)^2 \mathcal{D}^{(j)}} \quad (17)$$

$$T_{5b}^{(n+1)} = - \frac{e^2}{(2\pi)^3} \int \frac{d^3 q}{2q^+} \frac{\bar{u}(p, s) \not{\epsilon}(q) (\not{P} + m) \not{\epsilon}(q) (\not{p}' + m) \mathcal{M}_n^{(j)} u(p, s)}{(p \cdot q)(p^- - p'^-) \mathcal{D}^{(j)}} \quad (18)$$

$$T_{5c}^{(n+1)} = \frac{e^2}{(2\pi)^3} \int \frac{d^3 q}{2q^+} \frac{\bar{u}(p, s) \mathcal{M}_n^{(j)}(\not{P} + m) \not{\epsilon}(q) u(p, s)}{(p \cdot q) \mathcal{D}^{(j)}} \quad (19)$$

where $P = p - q$ and $p' = p$.

There are additional contributions in $(n+1)^{th}$ order, when we use the coherent state basis which are shown in Fig. 6 and are given by

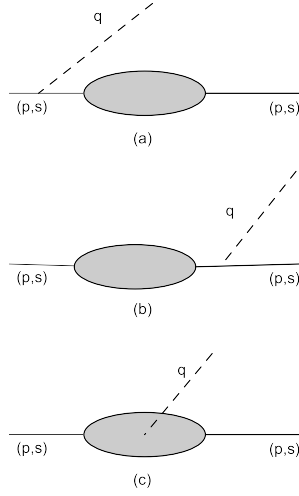


Fig. 6 Addition of a photon line to $O(e^{2n})$ blob in coherent state basis

$$T_{6a}'^{(n+1)} = - \frac{e^2}{(2\pi)^3} \int \frac{d^3q}{2q^+} \frac{\bar{u}(p,s) \mathcal{M}_n^{(j)}(\not{P} + m) \not{\epsilon}(q) u(p,s) (p \cdot \epsilon(q)) \Theta_\Delta(q)}{(p \cdot q)^2 \mathcal{D}^{(j)}} \quad (20)$$

$$T_{6b}'^{(n+1)} = \frac{e^2}{(2\pi)^3} \int \frac{d^3q}{2q^+} \frac{\bar{u}(p,s) \not{\epsilon}(q) (\not{p}' + m) \mathcal{M}_n^{(j)} u(p,s) (p \cdot \epsilon(q)) \Theta_\Delta(q)}{(p \cdot q) (p^- - p'^-) \mathcal{D}^{(j)}} \quad (21)$$

$$T_{6c}'^{(n+1)} = - \frac{e^2}{(2\pi)^3} \int \frac{d^3q}{2q^+} \frac{\bar{u}(p,s) \mathcal{M}_n^{(j)} u(p,s) (p \cdot \epsilon(q)) \Theta_\Delta(q)}{(p \cdot q) \mathcal{D}^{(j)}} \quad (22)$$

In the limit, $q^+ \rightarrow 0$, $\mathbf{q}_\perp \rightarrow 0$, $\not{P} \not{\epsilon}(q) \rightarrow p \cdot \epsilon$. As a result, the IR divergences sum of Eqs. (17) – (19) are exactly cancelled by the sum of Eqs. (20) – (22) leading to an infrared finite result.

4 Conclusions

We have demonstrated, using the coherent state formalism, that the true IR divergences in self energy correction cancel to all order in LFQED.

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